7èmes Journées Scientifiques de l'Université de Toulon - Palais NEPTUNE

(9 et 10 avril 2013)

Propagation des Ondes en Milieux Complexes - Wave Propagation in Complex Media

Radjesvarane ALEXANDRE

Title: **Quelques modèles cinétiques pour la condensation de Bose Einstein**
Abstract: nous présenterons quelques modèles basés sur des équations cinétiques pouvant décrire la condensation de Bose Einstein. Certains de ces travaux ont été faits en collaboration avec Jie LIAO et Chunjin LIN (Shanghai, PRC China).

Marc BRIANE

Title: **Caractérisation des champs électriques dans les composites périodiques**
Abstract: Dans ce travail en collaboration avec G.W. Milton et A. Treibergs de l'université de l'Utah, on cherche à caractériser parmi tous les gradients vectoriels périodiques réguliers ceux qui sont des champs électriques isotropiquement réalisables c'est-à-dire solutions d'une équation de conduction avec une conductivité isotrope convenable. En toute dimension une condition suffisante de réalisabilité est que le gradient ne s'annule pas dans l'espace. Cette condition est aussi nécessaire en dimension 2 mais pas en dimension trois. Cependant, lorsque l'on impose à la conductivité d'être aussi périodique la condition précédente n'est plus suffisante. On obtient alors, à l'aide du système dynamique associé au gradient, une condition nécessaire et suffisante de réalisabilité dans le tore qui est illustrée par un exemple. On traite également le cas de la réalisabilité d'un gradient matriciel périodique et le cas moins régulier des laminés.

Eric BONNETIER

Title: **Some properties of the spectrum of the Poincaré-Neumann operator in composite media with close-to-touching inclusions**
Abstract: The Poincaré-Neumann operator naturally appears in the integral formulation of elliptic transmission problems. This operator has received some attention in recent years in the context of metamaterials, and plasmonics. We study its spectrum in composite media containing close-to touching inclusions. In the narrow channels between such inclusions, the gradients of the solutions of the underlying PDE's may become large. In the case of disks, we relate the size of the gradients to the spectral properties of the integral operator. We also study the dependance of its eigenvalues on the inter-inclusion distance and on the coefficient contrast for C^{1+m} inclusions. This is joint work with Faouzi Triki (Université Joseph Fourier, Grenoble).

Mourad CHOULLI

Title: **Stability estimate for the determination of the time dependent external potential from the DN map in a periodic quantum waveguide.**
Abstract: In a recent work, in collaboration with Yavar Kian and Eric Soccorsi, we established a logarithmic stability estimate for the inverse problem consisting in the determination of the time dependent external potential from the DN map in a periodic quantum waveguide. We first reduce our problem, via a partial Floquet-Bloch-Gel'fand transform, to an inverse Schrödinger problem in a bounded domain with quasi-periodic boundary condition in one direction. Next, following an idea by Hhner, we

construct a family of complex geometric optic solutions. This family of particular solutions plays the key role in the proof our stability estimate.

Horia CORNEAN

Title: **Number of antennas versus scattering environment in MIMO systems**
Abstract: We study the capacity of a MIMO (Multiple Input Multiple Output) system using a deterministic description for the transfer matrix, when no channel knowledge is available at the transmitter side. The capacity is studied as the number of antennas in the arrays increases. In one scenario, the antennas are assumed to fill in a given, fixed volume. In the other one, the distance between the antennas is assumed to be constant. In both situations, the capacity grows much more slowly than linearly with the number of antennas and it can even saturate. The scattering properties of the environment are at least as important as the number of antennas in the capacity values.

This is joint work with Francois Bentosela (Marseille) and Nicola Marchetti (Dublin).

Didier FELBACQ

Title: **Electromagnetic waves propagation near resonances in metamaterials**
Abstract: The propagation of electromagnetic waves in a medium made of parallel dielectric rods is studied in the vicinity of the Mie resonances of the rods. It is shown that in the low frequency domain the Maxwell equations can be homogenized, which leads to an equivalent permittivity and more surprisingly to an effective permeability. It is demonstrated that the magnetic resonance is a switch for the existence of a high density of states. The effect of spatial dispersion and the properties of universality and scaling are also investigated.

Bernard HELFFER

Title: **Global stability of the normal state of superconductors in the presence of a strong electric current**
Abstract: We consider the time-dependent Ginzburg-Landau model of superconductivity in the presence of an electric current flowing through a two-dimensional wire. We show that when the current is sufficiently strong the solution converges in the long-time limit to the normal state. We provide two types of upper bounds for the critical current where such global stability is achieved: by using the principal eigenvalue of the magnetic Laplacian associated with the normal magnetic field, and through the norm of the resolvent of the linearized steady-state operator. In the latter case we estimate the resolvent norm in large domains by the norms of approximate operators defined on the plane and the half-plane. We also obtain a lower bound, in large domains, for the above critical current by obtaining the current for which the normal state looses its local stability.

After Yaniv Almog and Bernard Helffer.

Dirk HUNDERTMARK

Title: **Mathematical challenges from non-linear fiber optics**
Abstract: We describe some recent rigorous work on soliton-like pulses in dispersion managed optical fiber channels:

"Dispersion management" refers to the engineering of an optical fiber channel with alternating spans of positive (normal) and negative (anomalous) dispersion fiber (periodic or otherwise) in order to achieve greater stability, bandwidth etc of optical information transfer. This technology has lead to a 100fold increase in bandwidth in long-haul optical transmission lines over intercontinental distances and it is widely used commercially nowadays. The simplest mathematical model describing pulses in a glass-fiber cable is the scalar one-dimensional nonlinear Schrödinger equation with cubic nonlinearity. "Dispersion management" means that the coefficient of dispersion is a function of distance (e.g. periodic) along the fiber waveguide. To model dispersion managed fiber channels one also averages over one period, yielding the Gabitov-Turitsyn equation, which is a non-local version of the non-linear Schrödinger equation.

It is well known that with constant negative (i.e., anomalous) dispersion there are soliton-like localized solutions and, not much of a surprise, for dispersion managed systems if the dispersion is, on the average, anomalous then there are again stable solitons.

However, in physical experiments, as well as numerical studies, it has long been observed that one gets soliton-like localized solutions even for average dispersion equal to **zero** dispersion. This was a surprise, both physically and mathematically, because the conventional wisdom had been that solitons emerge from a combination of nontrivial linear dispersion and nonlinearity. Something more subtle is going on in the zero average dispersion case, which is also the most important case from an applications point of view.

Rigorous results on soliton-like pulses for the Gabtov-Turitsyn equation, the so-called dispersion management solitons, have been rare (I know of 7 or so), which is mainly due to its non-locality, which makes it hard to study. Rigorous results for zero average dispersion are even rarer, since this case is a singular limit. This is quite in contrast to the enormous amount of experimental, numerical and theoretical work (if one searches for "dispersion management" on Google scholar one gets roughly one million hits).

We will discuss recent work on the decay and regularity properties of dispersion management solitons. Our results include a simple proof of existence of solutions of the dispersion management equation under mild conditions on the dispersion profile, which includes all physically relevant cases, regularity of weak solutions, and most recently a proof of exponential decay of dispersion management solitons, which confirms the theoretically and experimentally seen fact that dispersion management solitons are very well-localized.

This is joint work with Burak Erdoğan, William Green, and Young-Ran Lee.

Édouard OUDET

Title: **Optimal tilings**
Abstract: The question of finding an optimal tiling with respect to some specific energy is very frequent in many physical domains like foams, micro-structure resistance, etc. We discuss in this talk the numerical approximation of such structures associated to geometrical and spectral energies

El Maati OUHABAZ

Title: **Autour de la Régularité maximale des équations d'évolution non-autonomes.**
Abstract: Cet exposé est centré autour du problème de la régularité maximale. En guise d'introduction nous passerons en revue certains résultats marquant pour des équations de type chaleur puis énoncer des résultats nouveaux concernant le cas des équations d'evolution non-autonomes (c-à-d les opérateurs considérés dependent du temps). Ce cadre présentent de grandes difficultés et des défis importants, certains problèmes sont ouverts depuis le debut des année 60. Nous verrons ensuite comment la régularité maximale peut s'appliquer pour l'étude de certaines équations quasi-linéaires avec des conditions au bord dynamiques.

Andrey PIATNITSKI

Title: **On the large time behaviour of solutions to the fractional diffusion equations.**
Abstract: The talk will focus on a fractional diffusion model with Caputo fractional-time derivative in a regular bounded domain. We will discuss the large time behaviour of solutions of the corresponding initial boundary problem.

Yannick PRIVAT

Title: **Optimisation de domaine pour l'observabilité d'EDP**
Abstract: Le but de cet exposé est d'étudier des problèmes d'optimisation de forme pour l'équation des ondes, de Schrödinger, ou de la chaleur sur un domaine Omega en dimension quelconque, avec des conditions frontières s'il y a un bord de type Dirichlet, Neumann, mixtes, ou Robin. Etant donné un

état initial, on peut observer la solution de l'équation sur un sous-ensemble omega de Omega, ou bien la contrôler vers l'équilibre (par exemple à l'aide de la méthode HUM), ou encore la stabiliser (par damping linéaire) avec un contrôle de support omega. Dans les trois cas, on se pose la question de déterminer quel est le "meilleur" domaine possible omega parmi tous les sous-ensembles de Omega de mesure donnée (disons $L*\mathrm{mes}(Omega)$ avec $0 < L < 1$). Ces questions sont d'abord étudiées à données initiales fixées, puis indépendamment des données initiales : par exemple, on se pose le problème de maximiser la constante d'observabilité parmi les domaines précédents. Il s'avère que ce problème est lié aux propriétés d'ergodicité quantique du domaine Omega, et notamment aux propriétés de type QUE (Quantum Unique Ergodicity).

Ce sont des travaux en collaboration avec E. Trélat (Univ. Paris 6) et E. Zuazua (BCAM, Bilbao, Espagne).

Semjon VUGALTER

Title: **Berezin-Li-Yau inequality with a correction term**

Abstract: The talk is based on a joint work with Hynek Kovařík and Timo Weidl.
Let Ω be an open bounded set in $\mathbb{R}^d$ and let $-\Delta$ be the Dirichlet Laplacian on Ω. We denote by λ_j the non-decreasing sequence of eigenvalues of $-\Delta$. The main object of our interest in this talk is the lower bound

$$\sum_{j=1}^{k} \lambda_j \geq \frac{d\, C_d}{d+2}\, V^{-\frac{2}{d}}\, k^{\frac{d+2}{d}}, \qquad C_d = (2\pi)^2 \omega_d^{-2/d},$$

where V stands for the volume of Ω and ω_d denotes the volume of the unit ball in $\mathbb{R}^d$, which is known as the Berezin-Li-Yau inequality.

We improve this inequality in dimension two by adding a positive correction term to its right-hand side. It is also shown that the asymptotical behaviour of the correction term is almost optimal. This improves a previous result by Melas.