Many Body Localization without quenched disorder

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Guiding question for us

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A': Yes-votes: Kagan-Maksimov (1984), Mueller-Schiulaz (2013), Garrahan et al. (2014), Grover and Fisher (2013), Altschuler??, Cirac??.

No-votes Huse-Nandkishore ??, me??

Plan of talk

- Recap (or intro) to Many-Body Localization (MBL) (with disorder): Heuristics and Imbrie's result.
- A model for MBL without disorder: Heuristic motivation.
- Analysis of resonant graph: Why the naive pro-loc argument is wrong!
- A rigorous result: Asymptotic localization.
- Strategy of Proof: Iterative Perturbation Theory (cfr. spectral flow, talk by Bach)

Quantum Spin Chains

- Hilbert space of N spins: $\mathcal{H}_N = \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \ldots \otimes \mathbb{C}^2$
- Spin 1/2 operators $S^{(i)}, i = 1, 2, 3$ on $\mathbb{C}^2$ and local copies $S_x^{(i)}$ on $\mathcal{H}_N$:

$$S_x^{(i)} = \ldots \otimes \underbrace{1}_{\text{site } x-1} \otimes \underbrace{S^{(i)}}_{\text{site } x} \otimes \underbrace{1}_{\text{site } x+1} \otimes \ldots$$

- Vectors $\Psi \in \mathcal{H}_N$. Example: classical configurations η : e.g. $|\uparrow\uparrow\uparrow\ldots\rangle$, $|\uparrow\downarrow\uparrow\downarrow\ldots\rangle$

$$\Psi_\eta = |\eta\rangle := \otimes_x |\eta(x)\rangle, \qquad \eta(x) \in \{\uparrow, \downarrow\}, \qquad S^{(3)}|\uparrow\rangle = |\uparrow\rangle$$

- Local Hamiltonians of the form

$$H = hH_{free} + tH_{hop},$$

h : field (will be site-dependent)
t : hopping strength

with

$$H_{free} = \sum_x S_x^{(3)}, \qquad H_{hop} = \sum_x \sum_{i,i'=1,2,3} J(i,i') S_x^{(i)} S_{x+1}^{(i')}$$

Think: a *generic* (non-integrable) local interaction.

Intuition for localization: Two-site model

$$H = h_1 S_1^{(3)} + h_2 S_2^{(3)} + t H_{hop}, \qquad h_1, h_2 \geq 0$$

$|\eta\rangle$ are $t = 0$ eigenstates. Do naive perturbation theory in t:

η	$\lvert\uparrow\uparrow\rangle$	$\lvert\uparrow\downarrow\rangle$	$\lvert\downarrow\downarrow\rangle$	$\lvert\downarrow\uparrow\rangle$
$E_{t=0}(\eta)$	$h_1 + h_2$	$h_1 - h_2$	$-h_1 - h_2$	$-h_1 + h_2$

If

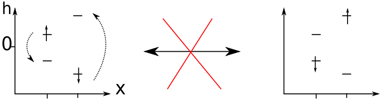
$$t \ll |\Delta E| = 2|h_1 \pm h_2|$$

then perturbation theory is good $\Rightarrow$ new eigenstates close to $t = 0$ eigenstates.

Perturbation theory in two-site model

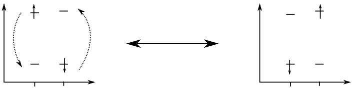
$$H = h_1 S_1^{(3)} + h_2 S_2^{(3)} + t H_{hop}, \qquad h_1, h_2 \geq 0$$

Localization if $t \leq |h_1 \pm h_2|$. Perturbation theory applies. Eigenstates at $t \neq 0$ small perturbations of $t = 0$.



$|\eta\rangle$'s do not hybridize (mix)

Delocalization if $t > |h_1 \pm h_2|$. PT does not apply. States η are resonant $\Rightarrow$ they mix and spread over two sites

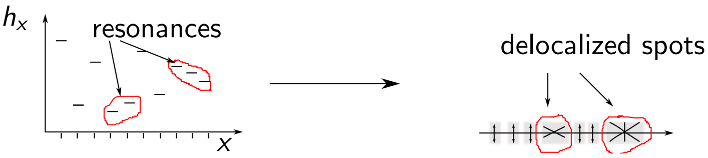


$|\eta\rangle$'s hybridize (mix)

Intuition: Full chain

$$H = \sum_x h_x S_x^{(3)} + tH_{hop}, \qquad h_x \text{ i.i.d. R.V.}$$

Bond $(x, x+1)$ resonant when $|h_x \pm h_{x+1}| \leq t$



If $t/Var(h) \ll 1$ (strong disorder) $\Rightarrow$ Resonant bonds are sparse and isolated. Try full PT

- away from resonant clusters: nondegenerate PT (large gaps): states remain local.
- at resonant clusters: degenerate PT: no control but clusters are themselves localised.

Iterative scheme (Imbrie-Spencer, Imbrie) provides a framework

Many Body Localization: towards precise meaning

Diagonalization: $\exists$ quasilocal unitary U such that

$$UHU^* = \tilde{H}_{free}, \qquad U \text{ locally close to } 1$$

- $\tilde{H}_{free}$ is diagonal in η-basis. $\tilde{H}_{free} = \sum_\eta \tilde{H}_{free}(\eta) \left|\eta\right\rangle\left\langle\eta\right|$.
- $\tilde{H}_{free}(\eta)$ is a random classical potential with exponential decay

$$\tilde{H}_{free}(\eta) = \sum_x f_x(\eta_x) + t f_{\{x,x\pm1\}}(\eta_x, \eta_{x\pm1}) + t^2 \ldots$$

...except at resonant spots,...where there is no spatial decay. Those resonant spots are rare with high Prob.

- Similar condition on U: $UO_xU^* = O'_x + tO'_{x,x\pm1} + \ldots$.
- All decays of course uniform in volume N.

Theorem (Imbrie)

$H = \sum_x (h_x S_x^{(3)} + t_x H_{hop,x})$ with i.i.d. disorder in both t_x, h_x.

Assumption: (Limited Level Attraction)

$$\mathbb{P}(\min_{E \neq E'} |E - E'| \leq \delta\} \leq \delta^\nu C^N$$

for some $C, \nu > 0$, and with E, E' e.v. of H_N (volume $[1, N]$).

$\Rightarrow$ MBL holds when $Var(t_x) \ll Var(h_x)$ (set $\mathbb{E}(t_x) = 0$)
What about assumption?

- OK if H_N has Poisson statistics,

$$\mathbb{P}(\min |E - E'| \leq \delta) \leq C\delta 4^N.$$

- OK if H_N has random-matrix statistics (level repulsion), then

$$\mathbb{P}(\min |E - E'| \leq \delta) \leq C\delta^2 8^N.$$

- No sensible reason to believe that e.v. would attract: $\nu < 1$.

MBL: Consequences

Diagonalization: $\exists$ quasilocal unitary U such that

$$UHU^* = \tilde{H}_{free}, \qquad U \text{ locally close to } 1$$

- η is a natural approx. and label for true eigenstates $U^*|\eta\rangle$.
- Full set of local commuting conserved quantitites $U^* S_x^{(3)} U$.
- Conductivity (defined as $N \to \infty$ by Green-Kubo) is zero.
- Order parameter

$$m := \frac{1}{2^N} \sum_{\text{eigenfunctions } \Psi} |\langle \Psi | S_x^{(3)} | \Psi \rangle|^2, \qquad \begin{array}{ll} m = 1 - \mathcal{O}(t) & \text{MBL} \\ m = \mathcal{O}(\mathrm{e}^{-CN}) & \text{ETH} \end{array}$$

because ETH (Eigenstate Thermalization Hypothesis):
$m \sim \langle \Psi | S_x^{(3)} | \Psi \rangle \approx \langle S_x^{(3)} \rangle_{Gibbs, T=\infty} = \frac{1}{2^N} \operatorname{Tr} S_x^{(3)} = 0,$
and MBL: $m \sim |\langle \eta | S_x^{(3)} | \eta \rangle| = 1$

How to make MBL without disorder?

Idea: Make systems with a large on-site space such that 'most' classical configs η appear disordered.

Example: Bose-Hubbard chain (with $q > 1$)

$$H = \sum_x h\, n^q(x) + t\, b(x) b^\dagger(x+1) + h.c., \qquad b(x), b^\dagger(x) \text{ bosons}$$

High density such that

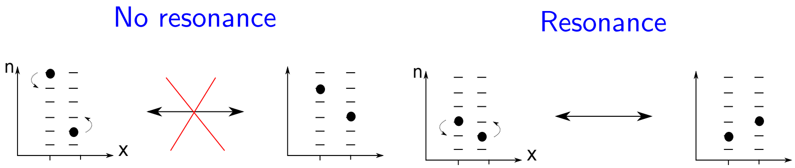
$$\nu := \langle n(x) \rangle_{\text{Gibbs}} \gg 1$$

$\sim$ cutoff for occupations $n(x) = b^\dagger(x) b(x)$ $\Rightarrow$ spin system with ν states/site.

Resonant spots

Let $t/h \ll 1$ (equivalent of strong disorder).

Anharmonicity of $n \to n^q$ gives us:



Only local swaps of levels $(n, n \pm 1)$ are resonant because

$$(\eta(x))^q + (\eta(x+1))^q = (\eta(x)-1)^q + (\eta(x+1)+1)^q$$

'*generically*' only if $\eta(x) - 1 = \eta(x+1)$.

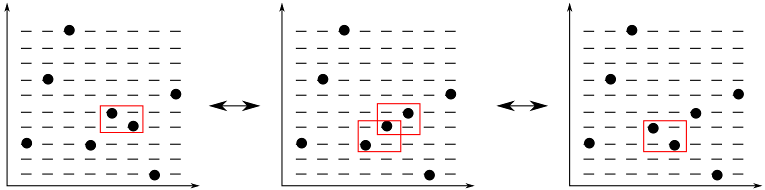
Resonant spots are rare for a typical config. η:

$$\mathrm{Prob}_\eta(\text{resonant spot at site } x) \sim 4/\nu$$

(Prob_η counting measure on η's. No disorder in model.)

But resonant spots can travel!

In disordered model: resonant spots defined from realization of disorder. **Here not!**

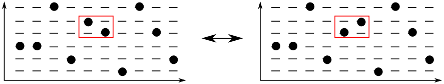


The resonant spot has shifted when moving (hybridizing) between resonant configs.

So if a typical η has only a few resonant spots, does it mean that it can hybridize only with a few other configs η'? (in disordered model: trivially 'yes')

Resonance graph $\mathcal{G}$

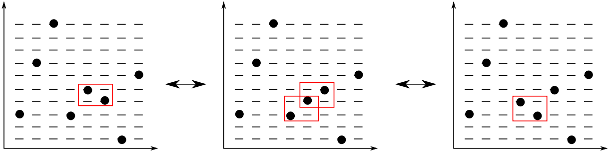
Def: $\eta \underset{\text{res}}{\sim} \eta'$ if connected by local swap $(n, n \pm 1) \to (n \pm 1, n)$.



$\eta \quad \underset{\text{res}}{\sim} \quad \eta'$

Def: Graph $\mathcal{G}$ on η's: (η, η') is edge iff. $\eta \underset{\text{res}}{\sim} \eta'$

Q: Small or large connected components?

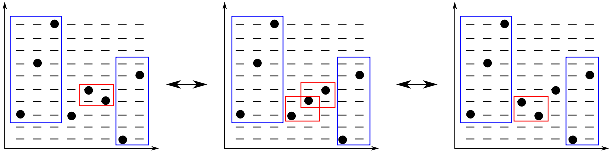


Example: Connected component of size 3 (total nb. configs $= \nu^8$)

Resonance graph $\mathcal{G}$: frozen sites

Def: Site x is frozen in config. η iff. $\eta'(x) = \eta(x)\ \forall \eta'$ in connected component of η.

Meaning: In PT, connected component of η is set of states with which η mixes (to produce perturbed eigenstate)
If x frozen in η, then perturbed eigenstate looks like η in x.



Example: 5 out of 8 sites are frozen.

Most sites are frozen

Answer: Take $\nu \gg 1$ and recall N is length chain.

$$\mathrm{Prob}_\eta(\text{at least } (1 - C/\nu)N \text{ sites are frozen}) \geq 1 - \mathrm{e}^{-cN}$$

So: Most perturbed eigenstates (in first order of PT) look locally like classic configs.

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But this is not at all robust!

Most sites are frozen . . . until we change the model

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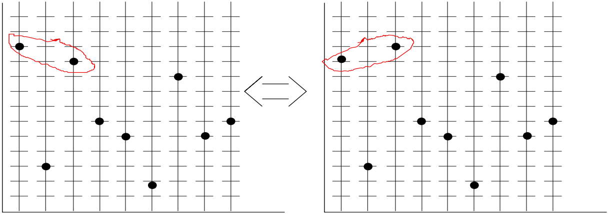
Small change in model: Add next-to-nearest neighbour hopping $\Rightarrow$ new, but similar, picture

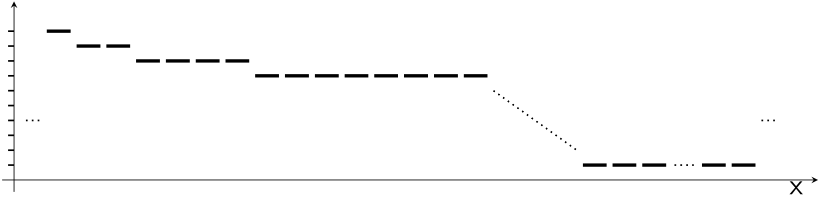


$$\mathrm{Prob}_\eta(\text{resonant spot at } x) \sim 8/\nu$$

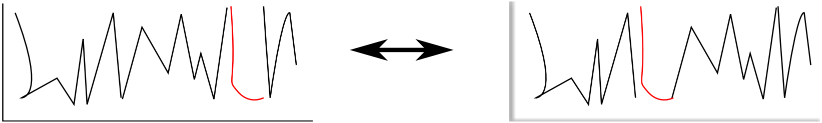
So, if $\nu \gg 8$, resonant spots are rare also within this model.

Resonant graph $\mathcal{G}$ nearly fully connected!

'Flemish Mountain' $\mathcal{F}$: Ultra-flat config of length 2^{ν}.



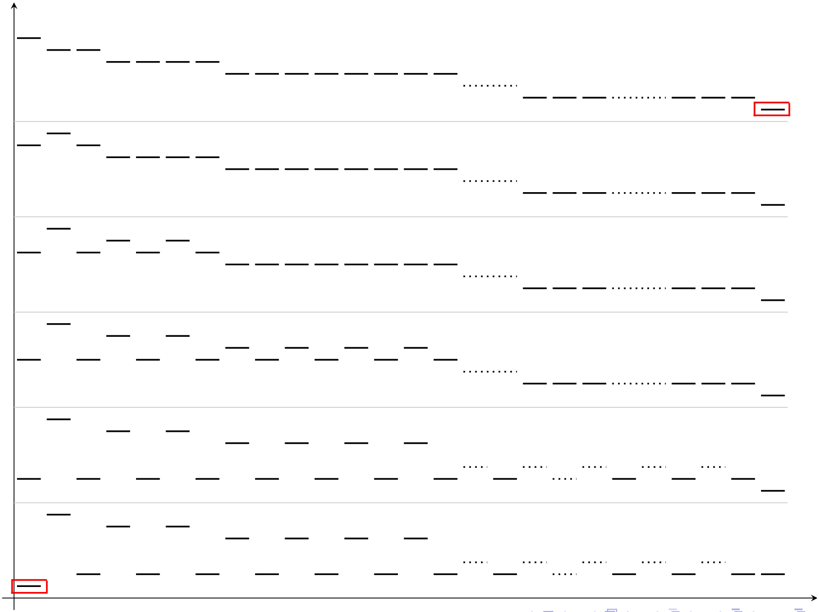
$\mathcal{F}$ (represented by) can travel through arbitrary background η:



$\mathcal{F}$ also helps to hybridize with all η''s and unfreezes all sites. Note: need volume at least $\sim \nu^{2^{\nu}}$ so that there is typically *somewhere* $\mathcal{F}$.

Mobility of $\mathcal{F}$

Transport of one level through $\mathcal{F}$.



What have we done?

Procedure

- Start with some $H = H_{free} + tH_{hop}$.
- Do 0'th order problem

$$P_{H_{free}=\mu N}\, H\, P_{H_{free}=\mu N} = Const + tA_{\mathcal{G}}, \qquad \mu \gg 1$$

with $A_{\mathcal{G}}$ a (weighted) adjacency matrix of graph $\mathcal{G}$.

'Result'

- $d = 1$ nearest neighbour hop.

$\Rightarrow$ $\mathcal{G}$ has many small components, size $\mathcal{O}(1) \Rightarrow A_{\mathcal{G}}$ manifestly loc.

- $d = 1$ next-to-nearest neighbour hopping or on two-lane strip
- $d > 1$.

$\Rightarrow$ $\mathcal{G}$ has a few large components, size $\mathcal{O}(CN) \Rightarrow$?
because $\exists$ connected graphs with adjacency matrix localised.

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Relevance: . . . and then, what about higher orders...?

Asymptotic Localisation

Idea: Localisation gets better as $\nu \to \infty$ and $T(\text{Temp}) \sim \nu^q$
$\Rightarrow$ Conjecture for conductivity $\kappa(T)$:

$$T^m \kappa(T) \to 0, \qquad \text{as } T \to \infty, \text{ for any } m > 0$$

This conjecture stands regardless of existence/non-existence $\mathcal{F}$ because $\mathrm{Prob}_\eta(\mathcal{F} \text{ appears at site } x) \sim \nu^{-2^\nu}$.

Def of conductivity κ : Split $H = \sum_x H_x$ and define current operator j_x:

$$j_x = \mathrm{i}[H, \sum_{y>x} H_x], \qquad \text{such that } \nabla j_x = -\partial_t j_x(t) = \mathrm{i}[H, H_x].$$

$$\text{Then} \qquad \kappa(T) := \frac{1}{T^2} \int_{-\infty}^{\infty} \mathrm{d}t \lim_{N\to\infty} \sum_x \langle j_x(t) j_0(0) \rangle_{\mathrm{Gibbs}_T}$$

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Too hard, ... but modify $\kappa(T) \to \kappa_\tau(T)$ by $\int_{-\infty}^{\infty} \to \int_{-\tau}^{\tau}$ (cutoff τ):

Asymptotic Localisation: Theorem

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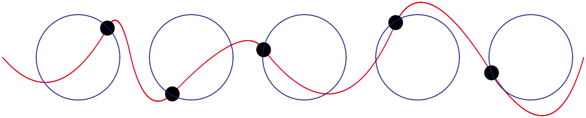
Theorem for $q > 2$: *almost* Asymptotic Loc

$$T^m \kappa_{\tau=T^p}(T) \to 0, \qquad \text{as } T \to \infty, \forall m > 0, p > m + C.$$

- Cutoff τ: Should model the fact that eventually the current-current correlation decays (so: *assumption* that there is no localization, but chaos)
- This suggests that transport has a non-perturbative origin in $1/T$.
- Asymptotic localisation is also what remains of localisation in disordered classical systems. (Dhar-Lebowitz 2008, Oganesyan-Pal-Huse 2009, Basko 2011). Here, no *real* loc expected.
- Asymptotic localisation ≈ Nekoroshev estimates in many body systems.

Asymptotic localization: Classical Mechanics

Rotor Chain



$$H(q,\omega) \;=\; \sum_x \left\{\frac{\omega_x^2}{2} - t\cos(\theta_x - \theta_{x+1})\right\} \quad \text{with} \quad \theta_x \in S_1, \dot{\theta}_x = \omega_x$$

Stochastic velocity flips $\omega_x \to -\omega_x$ with rate δ s.t. full dynamics

$$\{H,\cdot\} \longrightarrow \{H,\cdot\} + \delta\,\text{Markov-Generator}_{\omega-\text{flips}}$$

Asymptotic Localization - alternative formulation

$$\kappa(t,\delta = t^p) \leq C(m)t^m, \qquad \forall m>0, p>m+C.$$

Very suggestive: Conductivity due to noise $\Rightarrow \kappa(t,0) \leq C(m)t^m$

Proof (Quantum case): Spectral Flow

Scheme (Imbrie, Spencer): Write $H = hH_{free} + tH_{hop} =: D + \epsilon V$

- In $H = D + \epsilon V$, we try to eliminate ϵV by unitary conjugation

$$\mathrm{e}^{-\epsilon A} H \mathrm{e}^{\epsilon A} = D - \epsilon[A, D] + \epsilon V + \mathcal{O}(\epsilon^2)$$

- Terms of $\mathcal{O}(\epsilon)$ indeed vanish if

$$\langle \eta', A\eta \rangle = \frac{\langle \eta', V\eta \rangle}{D(\eta') - D(\eta)}$$

  with $D(\eta) = \langle \eta, D\eta \rangle$.

- Hence split $V = V_{res} + V_{Nres}$ with

$$V_{res} = \sum_{\eta,\eta'} \chi[D(\eta) - D(\eta') = 0]\ |\eta\rangle\langle\eta V \eta'\rangle\langle\eta'|$$

We can only eliminate V_{Nres} because division by $D(\eta') - D(\eta)$

Resonant Hamiltonian

We obtain unitarily equivalent Hamiltonian

$$H^{(1)} := \mathrm{e}^{-\epsilon A} H \mathrm{e}^{\epsilon A} = D + \epsilon V_{res} + \mathcal{O}(\epsilon^2)$$

- The $\mathcal{O}(\epsilon^2)$ term is the new perturbation $\Rightarrow$ do again rotations to get $\mathcal{O}(\epsilon^4), \mathcal{O}(\epsilon^8), \ldots$. This decreases the 'hopping' term! $\Rightarrow$ decreases transport.
- However, resonant part ϵV_{res} has to be treated nonpeturbatively. We need that $D + \epsilon V_{res}$ is localized. $\Rightarrow$ analysis of resonance graph $\mathcal{G}$.

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Thanks